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# METHOD AND SETUP FOR MEASUREMENT OF DIELECTRIC PERMITTIVITY IN WIDE FREQUENCY RANGE

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The primary goal of this work is to measure dielectric constant of ferrite material in the frequency range from ~ 300 kHz to ~ 20 MHz. Rectangular (1" x 3") shaped samples of ferrite (t = 0.2") have been used for this work. Several different methods can be used; the simplest one was chosen that is based on the capacitance measurement.

## 1. Method Description

A simplified schematic of the measurement setup is shown in Fig. 1.

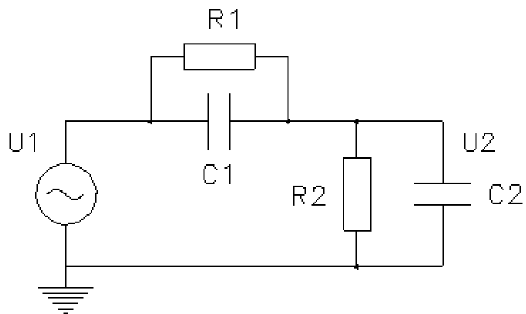



Fig. 1: Schematic of capacitance measurement setup

AC source with regulated frequency feeds a simple circuit of a capacitance C1 (to be measured) and a resistance R2. During measurement, a measurement device is represented by a capacitor C2 (input resistance of the device in very high). Power loss in C1 is represented by R1. In the simplest case when C2 << C1 and R2 << R1 (or C2R2 << C1R1), the amplitude and the phase of the circuit transfer function **k = U2/U1** can be written as

$$\left|\frac{U2}{U1}\right| = \frac{\omega \cdot R2 \cdot C1}{\sqrt{1+(\omega \cdot R2 \cdot C1)^2}}$$
$$tg\theta = \frac{1}{\omega \cdot R2 \cdot C1} \tag{1}$$

**If a product R2·C1 is small** compared with the period of oscillation (low frequency), the phase of the output voltage U2 is ahead of the input voltage by ~90°. Sensitivity in this case is small. As we try to increase the sensitivity by increasing R2, the phase gradually moves towards zero.

We can find C1 independently by measuring the amplitude and the phase of the transfer function:

$$\frac{1}{C1} = \omega \cdot R2 \cdot \sqrt{\frac{1}{k^2} - 1}\,; \qquad \frac{1}{C1} = \omega \cdot R2 \cdot tg\theta\,. \tag{2}$$

We can use this approximation in the lower part of the required frequency range.

**In a more common case** we can not neglect R1 and C2, especially when it is desirable to increase sensitivity of the scheme without using additional wide band amplifiers. Then we can write down the next expression:

$$\left|\frac{U2}{U1}\right| = \sqrt{\frac{(1/R1)^2 + (\omega C1)^2}{(1/R1 + 1/R2)^2 + \omega^2 \cdot (C1+C2)^2}} \qquad (3)$$

$$tg\theta = \frac{\omega \cdot (C1R1 - C2R2)}{1 + R2/R1 + \omega^2 \cdot R1R2 \cdot C1 \cdot (C1+C2)}$$

This expression converges to (1) if R2C2 << R1C1 and R1→ ∞. The phase can be positive and negative depending on the relation between the elements of the scheme. Here we have two unknowns: C2 and R1. Having two expressions for two unknowns make it possible to find both C1 (and then dielectric constant of the material; see below) and R1 (and the loss tangent of C1) if we know R2 and C2.

**2. Setup Description**

As an AC source, 8116 A HP function generator was used. With the amplitude of the output voltage (U1) of ~ 1.5 V, it is capable to change frequency up to 50 MHz. The output impedance of the generator is ~ 50 Ohm. As a measuring device, 199C FLUKE scope was used with the bandwidth of 200 MHz. and the maximum sensitivity of ~20mV. Input resistance of the oscilloscope is 1 MOhm and input capacitance is 14 pF. Expected capacitance of a capacitor made by adding two 20 x 70 mm electrodes to the sample of the ferrite material is ~ 20pF; to increase sensitivity of the setup and to arrange for a better protection against stray electrical field, the sample composite capacitor is built by connecting two elementary capacitors in parallel as it is shown in Fig. 2.

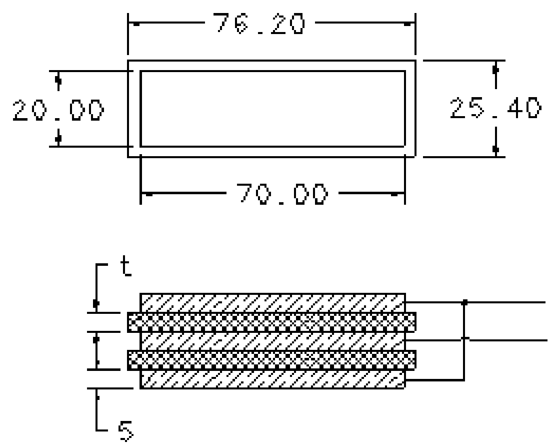



Fig. 2: Composite sample capacitor C1 (dimensions in mm)

The central electrode is connected to the power source and the two peripheral electrodes are connected to the output resistance R2 (see schematic in Fig. 1), which should be stable within the required frequency range. Film-type 0.125 W resistor was chosen here with the measured resistance R2 = 148 Ohm.

Two types of probes connecting the output point to the oscilloscope were used. The first probe was just a piece of a cable with the measured capacitance of 120 pF (820 Dynascan capacitance meter). With the input capacitance of the scope, this makes C2 = 134 pF, R2 = 148 Ohm. The second probe was a 10:1 probe with the input impedance of (10MOhm, 14 pF) when connected to the scope. This makes R2 = 148 Ohm and C2 = 15 pF. Because the probes had different lengths, (~ 7.5 ns more delay for the 1:1 probe), it was necessary to adjust for the additional phase gain at every frequency.

### 3. Testing the setup

With the measurement method chosen and C2 and R2 defined, measurement of two samples of different dielectric materials was conducted. The first material was G-10 plate with thickness $t_{G10}$ = 1.58 mm; the second material was Teflon plate with $t_{Tefl}$ = 3.25 mm. After the transition coefficient and the phase difference between the input and the output signals were measured using the setup described above, the corresponding values of the capacitance C1 was found by direct calculation using (2) or by using MathCad to solve the system of equation (3). In the result tables below you can also find the data of the measured phase that often indicate reliability of the obtained information.

#### 3.1 G-10 plate as an insulator

The capacitance of the system measured by the 820 Dynascan capacitance meter was 67.6 pF.

Table 1 shows values of the capacitance and the loss tangent measured using the measurement setup described earlier with the probe 10:1.

Table 1: Measurements results with the probe 10:1

| f (MHz) | 0.3 | 0.5 | 1.00 | 3.00 | 10.00 | 20.00 |
|---|---|---|---|---|---|---|
| C11 (pF) | 68.9 | 67.9 | 65.0 | 63.7 | 59.3 | 45.7 |
| Phase | 87 | 86 | 77 | 74 | 52 | 27 |
| C1 | 68.9 | 67.9 | 65.0 | 64.8 | 65.6 | 68.10 |
| tg(δ) | 0.029 | 0.014 | 0.15 | 0.063 | 0.028 | 0.021 |

Here C11 stands for the capacitance value calculated based on the measured data with the use of (2). For calculation of C1 and tg(δ) expression (3) is used. Corresponding graphs are shown in Fig. 3.

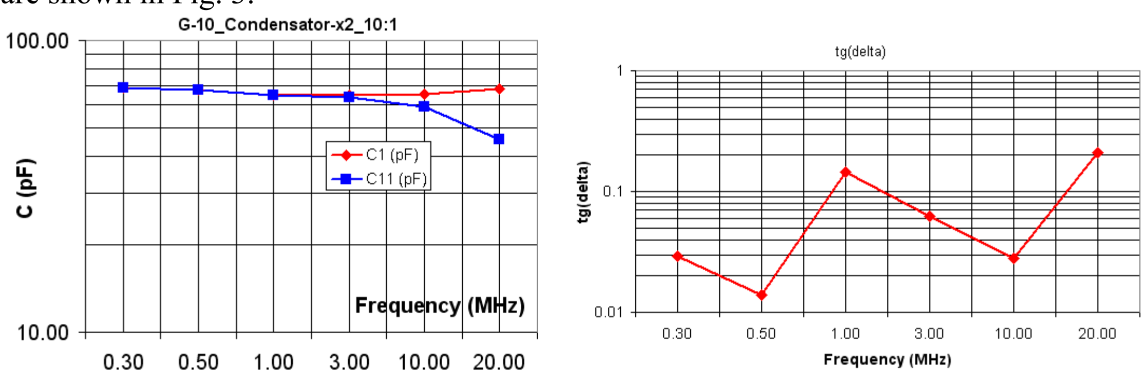



Fig. 3: Capacitance and loss tangent as a function of frequency. Data taken with the use of 10:1 probe.

Table 2 shows values of the capacitance and the loss tangent measured using the measurement setup described earlier with the probe 1:1.

Table 2: Measurements results with the probe 1:1

| f (MHz) | 0.3 | 0.5 | 1.00 | 3.00 | 10.00 | 20.00 |
|---|---|---|---|---|---|---|
| C11 (pF) | 67.6 | 66.5 | 63.9 | 62.8 | 62.6 | 43.0 |
| Phase (deg) | 88 | 84 | 79 | 61 | 26 | 16 |
| C1 | 67.7 | 66.2 | 63.0 | 56.0 | 32.3 | 13.50 |
| tg(δ) | 0.016 | 0.016 | 0.018 | 0.027 | 0.042 | 0.062 |

Corresponding graphs are shown in Fig. 4.

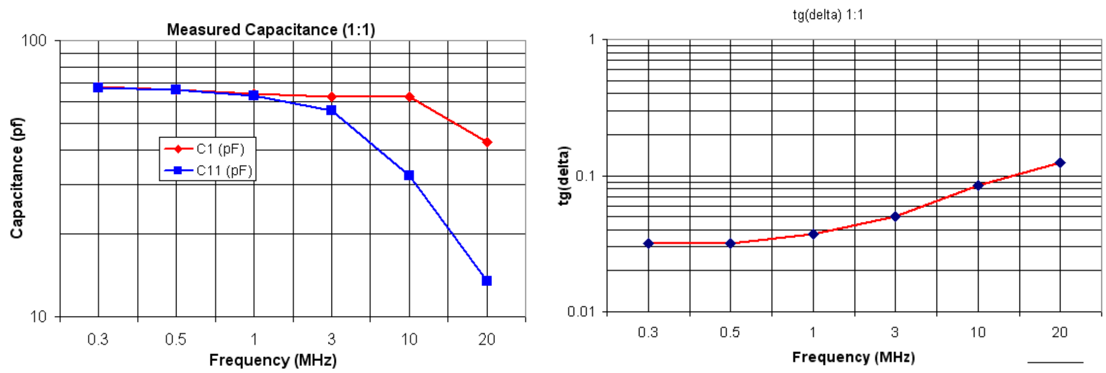



Fig. 4: Capacitance and loss tangent as a function of frequency. Data taken with the use of 1:1 probe.

### 3.2 Teflon plate as an insulator

The capacitance of the system measured by the capacitance meter was 17.1 pF. Table 3 shows values of the capacitance and the loss tangent measured using the measurement setup described earlier with the probe 10:1.

Table 3: Measurements results with the probe 10:1

| f (MHz) | 0.5 | 1.00 | 3.00 | 10.00 | 20.00 |
|---|---|---|---|---|---|
| C11 (pF) | 18.7 | 18.7 | 18.3 | 18.6 | 15.4 |
| Phase (deg) | 87 | 85 | 84 | 72 | 50 |
| C1 | 19.7 | 18.7 | 18.4 | 19.2 | 17.6 |
| tg(δ) | 0.037 | 0.057 | 0.015 | 0.016 | 0.19 |

Corresponding graphs are in Fig. 5

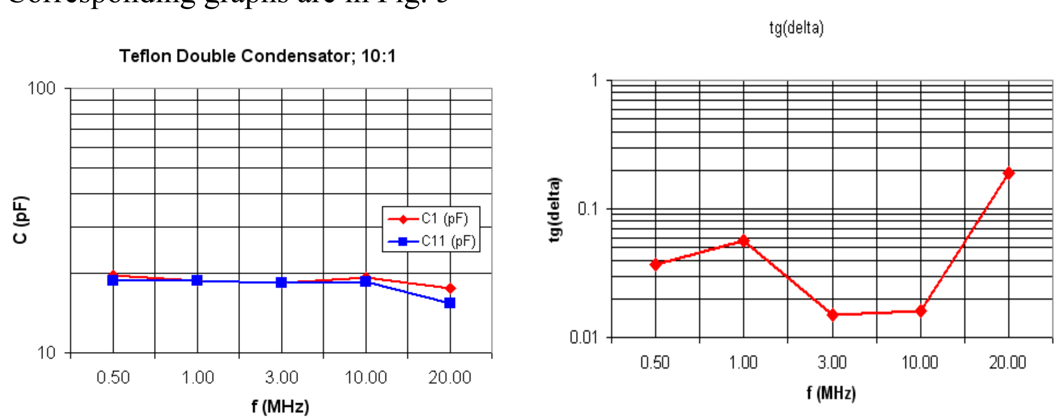


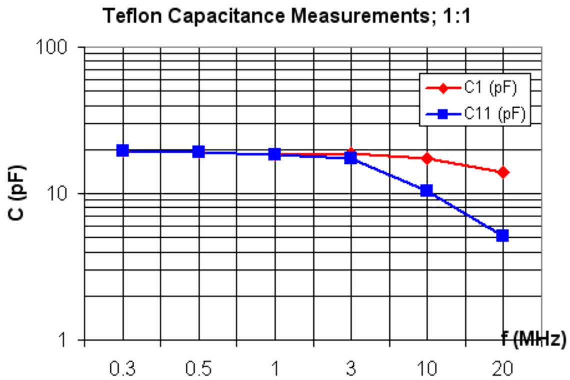

Fig. 5: Capacitance and loss tangent as a function of frequency. Data taken with the use of 10:1 probe.

Table 4 shows values of the capacitance and the loss tangent measured using the measurement setup described earlier with the probe 1:1.

Table 4: Measurements results with the probe 1:1

| f (MHz) | 0.3 | 0.5 | 1.00 | 3.00 | 10.00 | 20.00 |
|---|---|---|---|---|---|---|
| C11 (pF) | 19.5 | 19.3 | 18.4 | 17.3 | 10.5 | 5.1 |
| Phase (deg.) | 89 | 85 | 80 | 67 | 36 | 19 |
| C1 | 19.5 | 19.3 | 18,5 | 18.6 | 17.3 | 14 |
| tg(δ) | 0.047 | 0.055 | 0.042 | 0.021 | 0.02 | 0.04 |

Corresponding graphs are shown in Fig. 6.



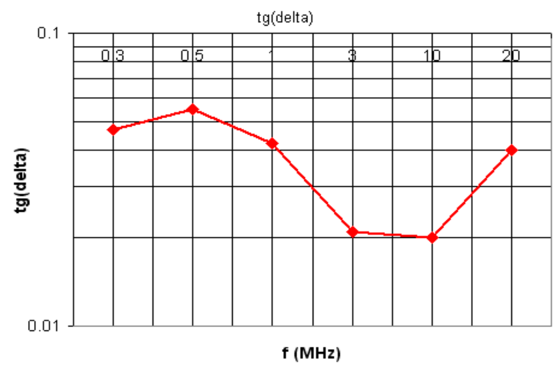




Fig. 6: Capacitance and loss tangent as a function of frequency. Data taken with the use of 1:1 probe.

## 4. Discussion

There is some difference in the results obtained by using different probes that increases with the frequency. Because in the case of 1:1 probe the capacitance of the cable can change with frequency, I find more reliable the data obtained with the use of 10:1 probe. In principle, it is possible to measure frequency dependence of the cable's capacitance, but at the moment I will think of it as of a second order effect.

Data for C1 in Fig 3 and Fig. 5 as representing the measured capacitance. If do not take into the account edges of the package, we can extract a value of the dielectric constant for the used insulating material, which is ~ 4.63 for G-10 (a range of dielectric constants for G10 found in handbooks is from ~4.5 to ~5.0) and ~2.6 for Teflon (the handbook value of 2.1). To take into the account the edges, some modeling work ought to be made. A quarter of a double capacitor with Teflon insulation is shown in Fig. 7

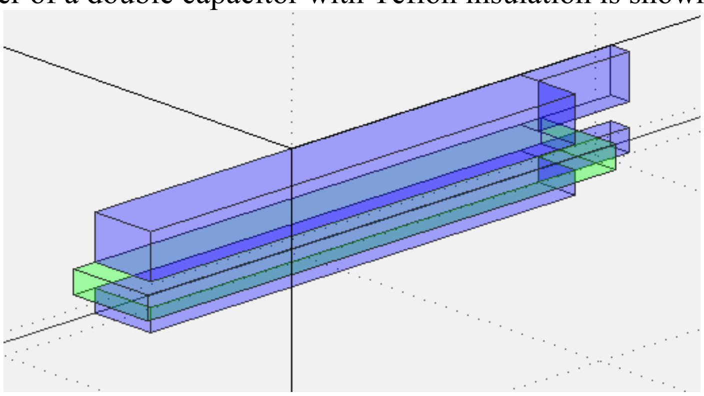


Fig. 7: 3D modele of the capacitor (1/4 of the capacitor is shown)

During the modeling, the total capacitance of the system is calculated as a function of the dielectric constant. Then, having this function available, a value of the dielectric constant is chosen that provides the value of the capacitance that agrees with the measurement results.

At ε= 2, we have the next distribution of partial capacitances in the whole system:
$C_\Sigma = 4 *\{(3697 +103 +61) + (421 + 68 + 54 + 28 + 19)\}*10^{-15}$ F =
$= 4*4.451*10^{-12}$ F = 17.8 pF

Here the sum inside the first couple of parenthesis corresponds to the surface facing Teflon. Inside the second pair of parenthesis are the values of the partial capacitances that correspond to the surface facing air; this surface accounts for ~ 15% of the total capacitance. Table 5 below shows these partial capacitances for the values of the dielectric permittivity of 1.5, 2.0, and 2.5.

Table 5: Measurements results with the probe 10:1

| | $C_{FACE}$ (pF) | $C_{AIR}$ (pF) | $C_\Sigma$ (pF) |
|---|---|---|---|
| ε = 1.5 | 11.28 | 2.71 | 13.99 |
| ε = 2.0 | 15.03 | 2.77 | 17.80 |
| ε = 2.5 | 18.78 | 2.81 | 21.59 |

It is possible to see that partial capacitances associated with the surfaces facing the air does not change much as the dielectric permittivity changes. A graph in Fig. 8 gives the dependence of the total capacitance on the permittivity.

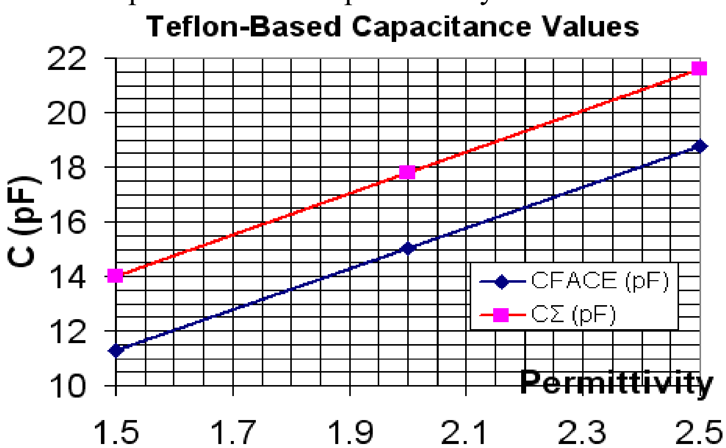



Fig. 8 Capacitance as a function of permittivity.

Based on the tables with the measured data and using the graph in Fig. 8, we can derive the data (see Table 6) showing the permittivity change in the used frequency range:

Table 6: Measurements results with the probe 10:1

| | 0.3 MHz | 0.5 MHz | 1 MHz | 3 MHz | 10 MHz | 20 MHz |
|---|---|---|---|---|---|---|
| ε_10_ϕ | | 2.25 | 2.12 | 2.08 | 2.17 | 1.975 |
| ε_10_0 | | 2.12 | 2.12 | 2.075 | 2.10 | 1.69 |
| ε_1_ϕ | 2.22 | 2.20 | 2.085 | 2.10 | 1.94 | 1.5 |
| ε_1_0 | 2.22 | 2.20 | 2.08 | 1.94 | 1.35 | |

Not all the data in the table are trustable. The data taken with the 1:1 probe should only be trusted when a significant phase shift, close to 90°, between the input and the output signals is obvious. The data obtained by using a simplified method also can be trusted only when the phase shift is about 90°. Graph in Fig. 9 summarizes the results.

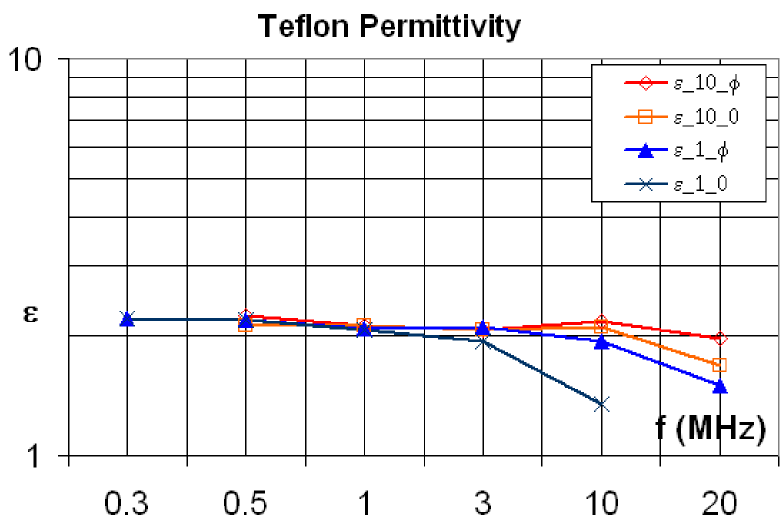



Fig. 9: Frequency dependence of the dielectric permittivity of Teflon insulator.

Now we can see that the permeability of Teflon is almost constant in the frequency range from 300 kHz up to 20 MHz. Possible deviation is about ±4% up to 10 MHz. There is some drop of the permittivity at 20 MHz, although some defects of the used method could be in charge here.

## 5. Conclusion

The method and the measurement setup described above was applied for measurement of dielectric permittivity of ferrite samples and gave reliable results consistent with what was expected (see TD-07-014).